Numerical Filling Simulation of Injection Molding Using Three-Dimensional Model

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Abstract Most injection molded parts are three-dimensional, with complex geometrical configurations and thick/thin wall sections. A 3D simulation model will predict more accurately the filling process than a 2.5D model. This paper gives a mathematical model and numeric method based on 3D model, in which an equal-order velocity-pressure interpolation method is employed successfully. The relation between velocity and pressure is obtained from the discretized momentum equations in order to derive the pressure equation. A 3D control volume scheme is employed to track the flow front. Th e validity of the model has been tested through the analysis of the flow in cavity.

Key words three dimension, equal-order interpolation, simulation, injection molding

1 Introduction

During injection moulding, the rheological response of polymer melts is generally non-Newtonian and nonisothermal with the position of the moving flow front. Because of these inherent factors, it is difficult to analyze the filling process. Therefore, simplifications usually are used. For example, in middle-plane technique and dual domain technique [1], because the most injection molded parts have the characteristic of being thin but generally of complex shape, the Hele-Shaw approximation [2] is used while analyzing the flow, i.e. the variations of velocity and pressure in the gapwise (thickness) dimension are neglected. So these two techniques are both 2.5D mold filling models, in which the filling of a mold cavity becomes a 2D problem in flow direction and a 1D problem in thickness direction.

However, because of the use of the Hele-Shaw approximation, the information that 2.5D models can generate is limited and incomplete. The variation in the gapwise (thickness) dimension of the physical quantities with the exception of the temperature, which is solved by finite difference method, is neglected. With the development of molding techniques, molded

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parts will have more and more complex geometry and the difference in the thickness will be more and more notable, so the change in the gapwise (thickness) dimension of the physical quantities can not be neglected. In addition, the flow simulated looks unrealistic inasmuch as the melt polymer flows only on surfaces of cavity, which appears more obvious when the flow simulation is displayed in a mould cavity.

3D simulation model has been a research direction and hot spot in the scope of simulation for plastic injection molding. In 3D simulation model, velocity in the gapwise (thickness) dimension is not neglected and the pressure varies in the direction of part thickness, and 3D finite elements are used to discretize the part geometry. After calculating, complete data are obtained(not only surface data but also internal data are obtained). Therefore, a 3D simulation model should be able to generate complementary and more detailed information related to the flow characteristics and stress distributions in thin molded parts than the one obtained when using a 2.5D model (based on the Hele-Shaw approximation). On the other hand, a 3D model will predict more accurately the characteristics of molded parts having thick walled sections such as encountered in gas assisted injection molding. Several flow behaviors at the flow front, such as "fountain flow", which 2.5D model cannot predict, can be predicted by 3D model. Meanwhile, the flow simulation looks more realistic inasmuch as the overall analysis result is directly displayed in 3D part geometry or transparent mould cavity.

This paper presents a 3D finite element model to deal with the three-dimensional flow, which employs an equal-order velocity-pressure formulation method [3,4]. The relation between

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velocity and pressure is obtained from the discretized momentum equations, then substituted into the continuity equation to derive pressure equation. A 3D control volume scheme is employed to track the flow front. The validity of the model has been tested through the analysis of the flow in cavity.

2 Governing Equations

The pressure of melt is not very big during filling the cavity, in addition, reasonable mold structure can avoid overbig pressure, so the melt is considered incompressible. Inertia and gravitation are neglected as compared to the viscous force.

With the above approximation, the governing equations, expressed in Cartesian coordinates, are as following:

Momentum equations

$$\frac{\partial}{\partial x}(2\eta\frac{\partial u}{\partial x})+\frac{\partial}{\partial y}[\eta(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})]+\frac{\partial}{\partial z}[\eta(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z})]-\frac{\partial(P)}{\partial x}=0 \quad (1a)$$

$$\frac{\partial}{\partial x}[\eta(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})]+\frac{\partial}{\partial y}(2\eta\frac{\partial v}{\partial y})+\frac{\partial}{\partial z}[\eta(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z})]-\frac{\partial(P)}{\partial y}=0 \quad (1b)$$

$$\frac{\partial}{\partial x}[\eta(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z})]+\frac{\partial}{\partial y}[\eta(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y})]+\frac{\partial}{\partial z}(2\eta\frac{\partial w}{\partial z})-\frac{\partial(P)}{\partial z}=0 \quad (1c)$$

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1d}$$

Energy equation

$$\rho C_p \frac{\partial T}{\partial t} = \rho C_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right) + \frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right) + \eta\dot{\gamma}^2 \tag{1e}$$

where, x, y, z are three dimensional coordinates and u, v, w are the velocity components in the x, y, z directions. P, T, ρ and η denote pressure, temperature, density and viscosity respectively.

Cross viscosity model has been used for the simulations:

$$\eta = \frac{\eta_0(T,P)}{1+(\eta_0\dot{\gamma}/\tau^*)^{1-n}}$$

where $n, \dot{\gamma}, \tau^*$ are non-Newtonian exponent, shear rate and material constant respectively.

Because there is no notable change in the scope of temperature of the melt polymer during filling, Arrhenius model [5] for η_0 is employed as following:

$$\eta_0(T,P) = B\exp\left(\frac{T_b}{T}\right)\exp(\beta P)$$

where B, T_b, β are material constants.

3 Numerical Simulation Method

3.1 Velocity-Pressure Relation

In a 3D model, since the change of the physical quantities are not neglected in the gapwise (thickness) dimension, the momentum equations are much more complex than those in a 2.5D model. It is impossible to obtain the velocity-pressure relation by integrating the momentum equations in the gapwise dimension, which is done in a 2.5D model. The momentum equations must be first discretized, and then the relation between velocity and pressure is derived from it. In this paper, the momentum equations are discretized using Galerkin's method with bilinear velocity-pressure formulation. The element equations are assembled in the conventional manner to form the discretized global momentum equations and the velocity may be expressed as following

$$u_i = \tilde{u}_i - K_i^u \frac{\partial P}{\partial x} \tag{2a}$$

$$v_i = \tilde{v}_i - K_i^v \frac{\partial P}{\partial y} \tag{2b}$$

$$w_i = \tilde{w}_i - K_i^w \frac{\partial P}{\partial z} \tag{2c}$$

where

$$\widetilde{u}_i = (-\sum_{i \neq j} A_{ij}^x u_j - B_{ij}^x v_j - C_{ij}^x w_j) / A_{ii}^x$$

$$\widetilde{v}_i = (-\sum_{i \neq j} B_{ij}^y v_j - A_{ij}^y u_j - C_{ij}^y w_j) / B_{ii}^y$$

$$\widetilde{w}_i = (-\sum_{i \neq j} C_{ij}^z w_j - A_{ij}^z u_j - B_{ij}^z v_j) / C_{ii}^Z$$

the nodal pressure coefficients are defined as

$$K_i^u = (\int_V N_i \mathrm{d}V) / A_{ii}^x$$

$$K_i^v = (\int_V N_i \mathrm{d}V) / B_{ii}^y$$

$$K_i^w = (\int_V N_i \mathrm{d}V) / C_{ii}^z \tag{3}$$

where $A_{ij}^x, B_{ij}^x, C_{ij}^x, A_{ij}^y, B_j^y, C_{ij}^y, A_{ij}^z, B_{ij}^z, C_{ij}^z$ represent global velocity coefficient matrices in the direction of x, y, z coordinate respectively. K_i^u, K_i^v, K_i^w denote the nodal pressure coefficients in the direction of x, y, z coordinate respectively. The nodal values for K_i^u, K_i^v, K_i^w are obtained by assembling the element-by-element contributions in the conventional manner. N_i is element interpolation and i means global node number and j is, for a node, the amount of the nodes around it.

3.2 Pressure Equation

Substitution of the velocity expressions (2) into discretized continuity equation, which is discretized using Galerkin method, yields element equation for pressure:

$$\int_V [\frac{\partial N_i}{\partial x}(N_j K_j^u \frac{\partial N_k}{\partial x} P_k) + \frac{\partial N_i}{\partial y}(N_j K_j^v \frac{\partial N_k}{\partial y} P_k) + \frac{\partial N_i}{\partial z}(N_j K_i^w \frac{\partial N_k}{\partial z} P_k)]\mathrm{d}V =$$
$$\int_V (\frac{\partial N_i}{\partial x} N_j \widetilde{u}_j + \frac{\partial N_i}{\partial y} N_j \widetilde{v}_j + \frac{\partial N_i}{\partial z} N_j \widetilde{w}_j)\mathrm{d}V$$

The element pressure equations are assembled in the conventional manner to form the global pressure equations.

3.3 Boundary Conditions

In the cavity wall, the no-slip boundary conditions are employed, e.g.

$$u = v = w = 0 \qquad \widetilde{u} = \widetilde{v} = \widetilde{w} = 0;$$

$$K_i^u = K_i^v = K_i^w = 0$$

on an inlet boundary,

$$u = v = w = \text{given}$$

$$K_i^u = K_i^v = K_i^w = 0$$

3.4 Velocity Update

After the pressure field has been obtained, the velocity values are updated using ne pressure field because the velocity field obtained by solving momentum equations does r satisfy continuity equation. The velocities are updated using the following relations

$$u_i = \tilde{u}_i - \frac{1}{A_{ii}^x}\int_V N\frac{\partial P}{\partial x}\mathrm{d}V \qquad v_i = \tilde{v}_i - \frac{1}{B_{ii}^y}\int_V N\frac{\partial P}{\partial y}\mathrm{d}V \qquad w_i = \tilde{w}_i - \frac{1}{C_{ii}^z}\int_V N\frac{\partial P}{\partial z}\mathrm{d}V$$

The overall procedure for fluid flow calculations is relaxation iterative, as shown in Fig and the calculation is stable without pressure oscillation.

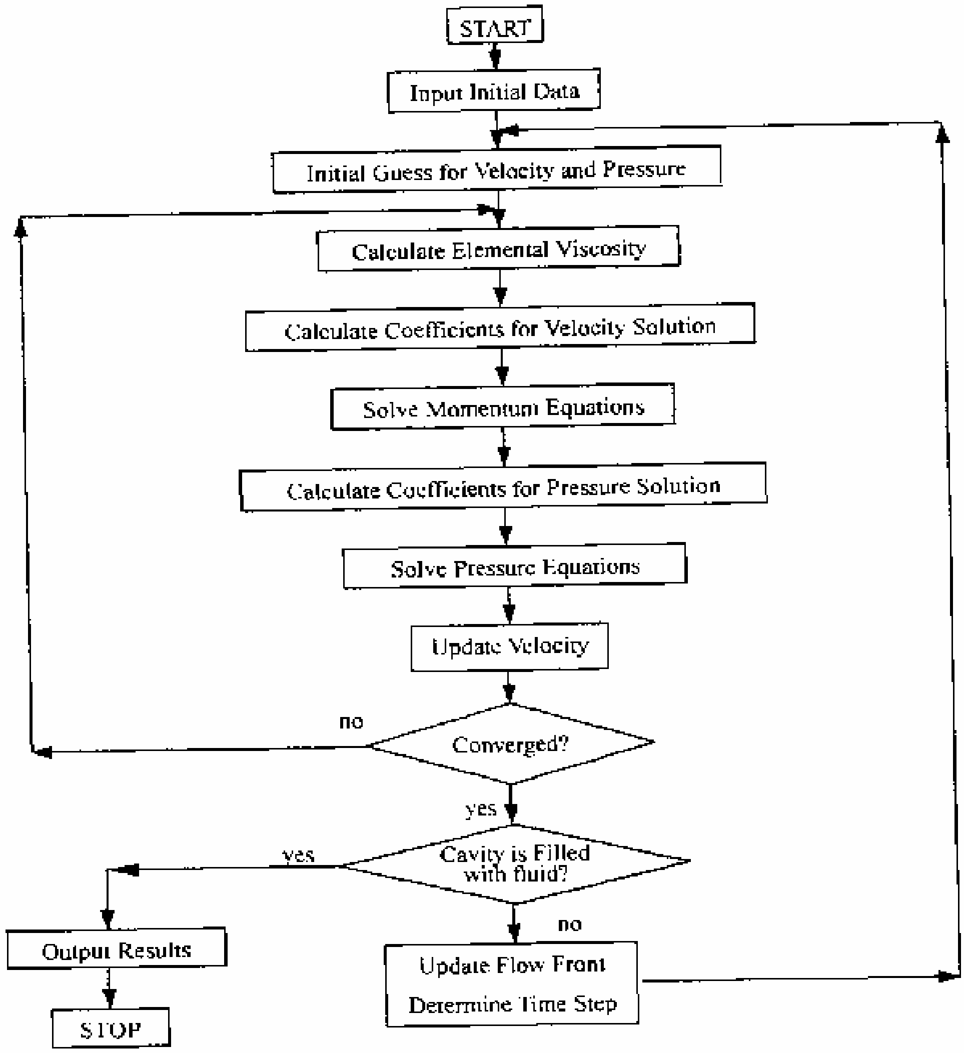
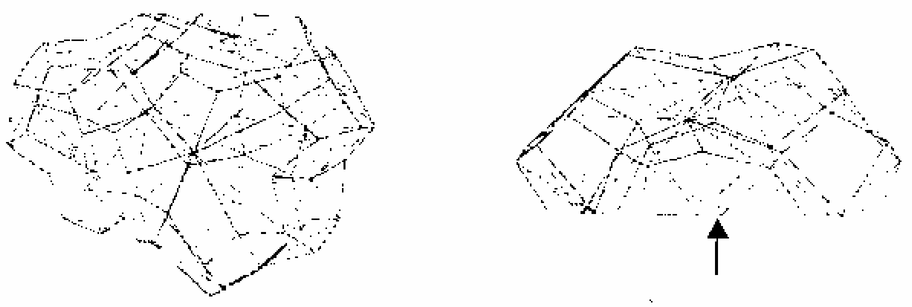


Fig.1 Flow chart used in three-dimensional simulation

3.5 The Tracing of the Flow Fronts

The flow of fluid in the cavity is unsteady and the position of the flow fronts varies with time. Like in 2.5D model, in this paper, the control volume method is employed to trace the position of the flow fronts after the FAN (Flow Analysis Network)[6]. But 3D control volume is a spacial volume and more complex than the 2D control volume. It is required that 3D control volumes of all nodes fill the part cavity without gap and hollow space. Two 3D control volumes are shown in Fig.2.

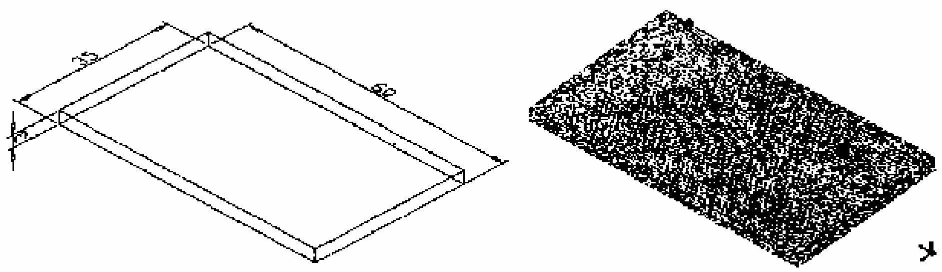


(a) control volume of an internal node (b) control volume of a boundary node

Fig.2 3D control volumes

4 Results and Discussion

The test cavity and dimensions are shown in Fig.3 (a). The selected material is ABS780 from Kumbo. The parametric constants corresponding to the n, τ^*, B, T_b and β of the five-constant Cross-type viscosity model are 0.2638, 4.514 $\times 10^4$ Pa, 3.13198043$\times 10^{-7}$ Pa*S, 1.12236 $\times 10^4$ K, 0.000 Pa^{-1}. Injection temperature is 45℃, mould temperature is 250℃, injection flow rate is 44.82 cu.cm/s. The meshed 3D model of cavity is shown in Fig.3 (b).

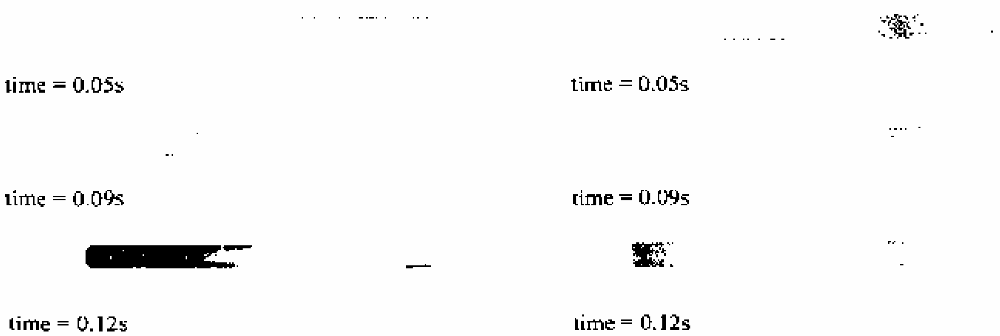


(a) the cavity dimension (b) the meshed cavity

Fig.3 Example part

"Fountain flow" is a typical flow phenomenon during filling. When the fluid is injected into a relatively colder mould, solid layer is formed in the cavity walls because of the diffusion cooling, so the shear near the walls takes place and is zero in the middle of cavity, and the fluid near the walls deflects to move toward the walls. The fluid near the center moves faster than the average across the thickness and catches up with the front so the shape of the flow front is round like the fountain. The round shape of the flow front of the example in several filling times predicted by present 3D model and shown in Fig.4(a), conforms to the theory and experiments. Contrarily, the shape of the flow front predicted by 2.5D model and shown in Fig.4(b) do not reveal the "Fountain flow".

The flow front comparison at the filling stage is illustrated in Fig.5. It shows that the predicted results based on present 3D model agree well with that based on Moldflow 3D model. The gate pressure is illustrated in Fig.6, compared with the prediction of Moldflow 3D model. It shows that the predicted gate pressure of present 3D model is mainly in agreement with that based on Moldflow 3D model. The major reason for this deviation is difference in dealing with the model and material parameters.

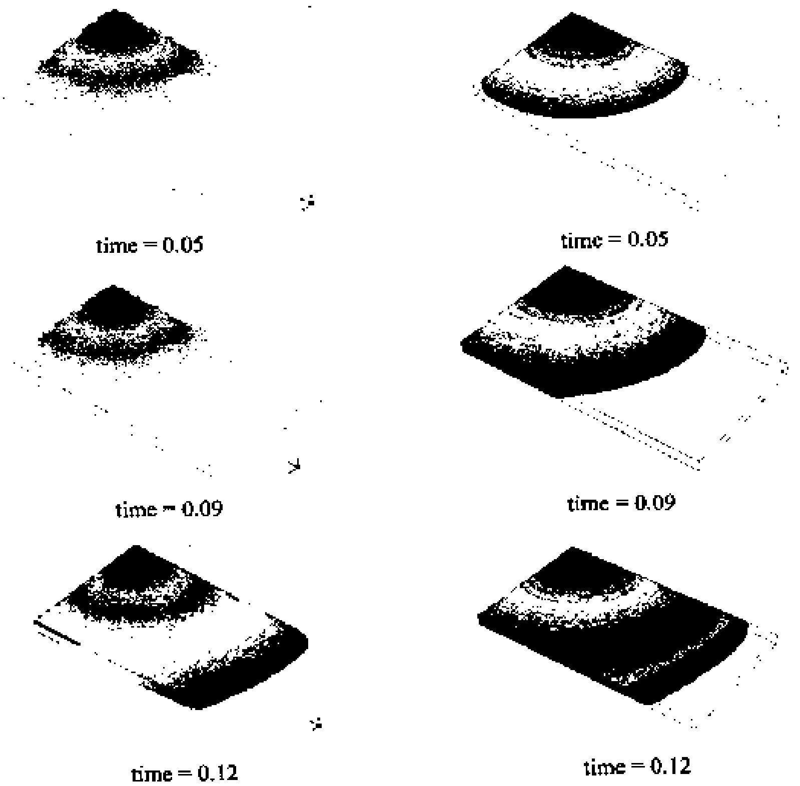


(a) shape of 3D flow front (b) shape of 2.5D flow front

Fig.4 Comparision between predicted shape of flow front based on present 3D model (a) and based on 2.5D model (b)

5 Conclusion

A theoretical model and numerical scheme to simulate the filling stage based on a 3D finite element model are presented. A cavity has been employed as example to test the validity. 3D numeral simulation of the filling stage in injection moulding is a development direction in the scope of simulation for plastic injection molding in the future. The long time cost is at present a problem for 3D filling simulation, but with the development of computer hardware and improvement in simulation technique, the 3D technique will be applied widely.



(a) (b)

Fig.5 Comparision between predicted flow front based on present 3D model (a) and based on Moldflow 3D model (b)

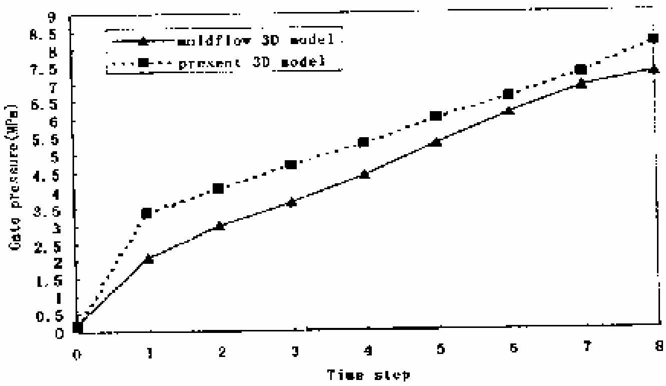


Fig.6 Comparison between predicted gate pressure based on present 3D model (dashed line) and based on Moldflow 3D model (solid line)

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